1 Preliminaries

- Launch Maple.
- Begin by clicking on the Text button in the context bar, and enter Your names, Math 227, Lab 2.
- From a new line, click on the Math button in the context bar.

2 Using Maple to find Area

We know that the signed area between the graph of \( y = f(x) \) and the \( x \)-axis on the interval \([a, b]\) is given by \( \int_a^b f(t) \, dt \). We can use this idea to find the area between two graphs \( y = f(x) \) and \( y = g(x) \). Suppose \( f(x) \geq g(x) \) for all \( x \) in the interval \([a, b]\). Then the area of the region between \( f(x) \) and \( g(x) \) on this interval is \( \int_a^b (f(x) - g(x)) \, dx \).

1. Find the area of the region bounded by \( y = x^2 \), \( y = \sqrt{x} \). You must first find the points of intersection of these graphs (to find \( a \) and \( b \)), then use the idea above.

2. The graphs of \( \sin(x) \) and \( \cos(x) \) interchange position at various points. For example, at \( x = 0 \), \( \cos(x) > \sin(x) \), but at \( x = \pi \), \( \sin(x) > \cos(x) \). Finding the area trapped between these two curves on an interval in which they interchange positions requires finding where the curves intersect inside your interval. Use this idea to compute the area between the graphs of \( y = \sin(x) \) and \( y = \cos(x) \) on the interval \([0, 2\pi]\).

3. Find the area under the curve \( y = x \) and above the \( x \)-axis on the following intervals

   (a) \([1, 2]\)
   (b) \([1, 10]\)
4. Find the area under the curve $y = x^2$ and above the $x$-axis on the following intervals
   (a) [1, 2]
   (b) [1, 10]
   (c) [1, 100]
   (d) [1, 1000]
   (e) [1, 1000000]

5. Comment on how your results for the previous two problems compare.

3 Using Maple to find the Length of Curves: Arclength

Suppose we wish to find the length of a curve $y = f(x)$ on the interval $[a, b]$. We can estimate this by partitioning the interval into $n$ equally sized subintervals, and drawing "secant" lines connecting endpoints on the curve over each subinterval. We will see in lab that the sum of lengths of these secant lines is given by

$$\sum_{i=1}^{n} \sqrt{1 + \left(\frac{f(x_i) - f(x_{i-1})}{\Delta x}\right)^2} \Delta x$$

As $\Delta x \to 0$, we see this converges to

$$\int_{a}^{b} \sqrt{1 + (f'(x))^2} \, dx$$

1. Determine each the arclength of the function on the specified interval.
   (a) $\sin(x)$ on $[0, \pi]$.
   (b) $4 - x^2$ on $[-2, 2]$
   (c) $\sqrt{4 - x^2}$ on $[-2, 2]$. Does this answer surprise you?